

Saturday, October 24, 2026, Eastern Zone

Time limit: 4 hours and 30 minutes.

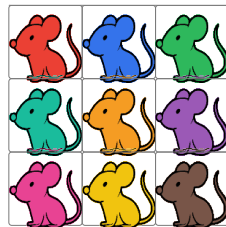
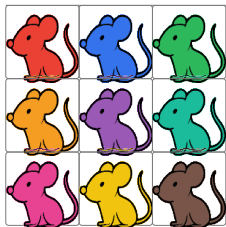
Each problem is worth seven points.

**Problem 1.** Let  $n$  be a positive integer that is not a perfect cube, and let  $a, b, c$  be nonzero integers such that  $a + b + c = 0$ . Prove that

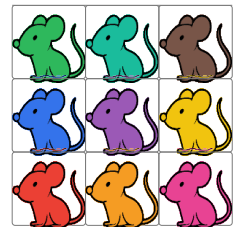
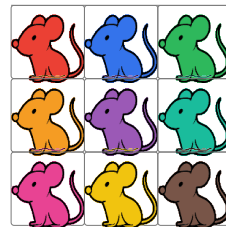
$$n(a^3 - n)(b^3 - n)(c^3 - n)$$

is **not** the cube of an integer.

**Problem 2.** Each cell of a  $2026 \times 2026$  grid has a different color. In one move, the colors in a single row or column may be shifted cyclically by any number of positions. Find the minimum number of moves needed to rotate the coloring by  $90^\circ$  counterclockwise.



Cyclic shift of the middle row.



Counterclockwise rotation by  $90^\circ$ .

**Problem 3.** Let  $ABC$  be a triangle with incenter  $I$  and circumcircle  $\Gamma$ . The incircle of triangle  $ABC$  touches sides  $AC$  and  $AB$  at points  $E$  and  $F$ , respectively. Let  $S \neq A$  be the second intersection point of  $\Gamma$  with the circumcircle of triangle  $AEF$ . Suppose the reflection of line  $SI$  across line  $EF$  meets lines  $AC$  and  $AB$  at points  $P$  and  $Q$ , respectively. Let  $X, Y$  be the points on line  $EF$  such that  $PX \perp AC$  and  $QY \perp AB$ . Prove that the circles with diameters  $CX$  and  $BY$  have a common point on line  $PQ$  and a common point on  $\Gamma$ .

