

Sunday, October 25, 2026, Western Zone

Time limit: 4 hours and 30 minutes.

Each problem is worth seven points.

Problem 4. Does there exist a geometric progression of positive integers $a_1, a_2, \dots, a_{2026}$ such that the sequence of their digit sums $S(a_1), S(a_2), \dots, S(a_{2026})$ forms an arithmetic progression with a non-zero common difference?

Here, $S(x)$ denotes the sum of digits of x in its decimal representation.

Problem 5. Let $\mathcal{P} = P_1P_2 \cdots P_{2026}$ and $\mathcal{Q} = Q_1Q_2 \cdots Q_{2026}$ be convex polygons such that every vertex of $\mathcal{Q}$ lies strictly in the interior of $\mathcal{P}$. Suppose that $Q_iQ_{i+1} \parallel P_iP_{i+1}$ for every $1 \leq i \leq 2026$, with subscripts taken modulo 2026.

Find the maximum number of indices $1 \leq i \leq 2026$ with $Q_iQ_{i+1} > P_iP_{i+1}$.

Problem 6. Fix an odd integer $k > 2026$. Yugi has a Yu-Gi-Oh! deck containing an odd number n of cards, each of which is one of two types: a *Monster Card* or a *Trap Card*.

On each move, Yugi chooses k distinct cards and uses the Millennium Puzzle to determine whether there are more Monster Cards or Trap Cards among those k cards. Cards may be chosen again on later moves, and his choices may depend on the answers he obtained on the previous moves.

Prove that, for all sufficiently large n , Yugi can determine which type is more numerous in the entire deck using fewer than $\frac{n}{100}$ moves.

Note. Results with $\frac{1}{100}$ replaced by a larger constant $c_k < 1$, which may depend on k , may receive partial credit depending on the value of c_k .

