

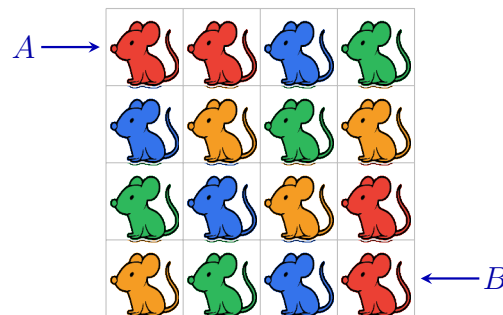
Saturday, October 24, 2026, Western Zone

Time limit: 4 hours and 30 minutes.

Each problem is worth seven points.

**Problem 1.** At the International Mouse Olympiad, 2026 teams of 2026 mice each are to occupy the cells of a  $2026 \times 2026$  grid, with one mouse per cell. To discourage collaboration within teams, the organizers wish to maximize the *team distance*: the sum of the taxicab distances over all unordered pairs of mice on the same team. What is the maximum possible team distance?

*Note.* If two mice occupy cells  $(r_1, c_1)$  and  $(r_2, c_2)$ , their taxicab distance is  $|r_1 - r_2| + |c_1 - c_2|$ .



In this sample  $4 \times 4$  arrangement, each colour represents a team of four. The red mice  $A$  and  $B$  contribute  $|1 - 4| + |1 - 4| = 6$  to the team distance. Summing over all same-colour pairs gives  $22 + 17 + 20 + 17 = 76$ .

**Problem 2.** Let  $\mathbb{N}$  be the set of positive integers. Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that

$$f^{f(n)+m}(n) = n + f(n + m)$$

holds for all  $n, m \in \mathbb{N}$ . Here,  $f^k(n)$  denotes  $\underbrace{f(f(\cdots f(n)\cdots))}_{k \text{ times}}$ .

**Problem 3.** Let  $A_1A_2A_3A_4A_5A_6$  be a convex hexagon such that the distance from  $A_i$  to the line  $A_{i-1}A_{i+1}$  is the same for all  $1 \leq i \leq 6$ , where indices are taken modulo 6. For each  $k \in \{1, 2\}$ , let  $I_k$  and  $G_k$  denote the incenter and centroid of triangle  $A_kA_{k+2}A_{k+4}$ , respectively. Suppose that  $I_1 \neq I_2$  and  $G_1 \neq G_2$ .

Prove that the lines  $I_1I_2$  and  $G_1G_2$  are either parallel or coincident.

