

Sunday, October 25, 2026, Eastern Zone

Time limit: 4 hours and 30 minutes.

Each problem is worth seven points.

Problem 4. Let B, C be fixed points in the interior of a circle ω . The line BC meets ω at points A, D such that A, B, C, D are collinear in that order. Let P be a point on ω , distinct from A and D . The lines PB and PC meet ω again at points X and Y , respectively. Suppose the common external tangents to the circumcircles of triangles ABX and CDY meet at point S . Prove that, as P varies, S lies on one of two fixed lines.

Problem 5. Let $\mathbb{R}_{>0}$ denote the set of positive real numbers and $\mathbb{R}$ the set of all real numbers. Find all strictly increasing functions $f : \mathbb{R}_{>0} \rightarrow \mathbb{R}_{>0}$ such that, for every $x, z > 0$, the equation

$$f(x^2 - xy + y^2) + f(y^2 - yz + z^2) = f(x^2 + xz + z^2)$$

has exactly one solution $y \in \mathbb{R}$.

Problem 6. Each positive integer is coloured with exactly one of the two colours *azure* and *beige*, and each colour is used infinitely often. Let

$$a_1 < a_2 < \cdots \quad \text{and} \quad b_1 < b_2 < \cdots$$

be the azure and beige integers, respectively. Form the real numbers

$$\alpha = 0.a_{b_1}a_{b_2}a_{b_3}\cdots, \quad \beta = 0.b_{a_1}b_{a_2}b_{a_3}\cdots,$$

where each decimal is formed by concatenating the usual decimal representations of the indicated integers. Can both α and β be rational?

